Interpretation of DAW Model Cavity Data Using Perturbation Theory

F. E. Mills

May 10, 1988

Introduction

A suitable theory of coupled cavities was developed by Nagle, Knapp, and Knapp[1]. We can use their results to interpret results from the DAW models. The results apply to a series of N+1 coupled resonators, the first and last of which are "half cells" which store only half the energy of the intervening cells. For perfectly constructed cells (all resonant frequencies ω_n and coupling constants k the same) the modes of oscillation of the chain are described by eigenvectors X^q where $X^q{}_n = \cos(\pi qn/N)$ for n,q = 0 to N. The frequency of mode q is given by:

$(1 - \omega_o{}^2/\omega_q{}^2) + k\cos(\pi q/N) = 0$. The important mode is the $\pi/2$ mode where q = N/2. The stored energy in cavity n is given by

$W(n)\, X_n{}^2/2$ where $W(n) = 1/2$, n = 0.N, and $W(n) = 1$ otherwise.

The eigenvectors are orthogonal in the sense that

$$\sum_{n=0}^{N} W(n)\, X^q{}_n\, X^p{}_n = \frac{N\,\delta(q-p)}{2W(q)}$$

If now the individual cavities have frequency errors, $\omega_o \rightarrow \omega_{on}$ and the shift in the frequency ω_q of mode q is given by

$$\frac{\delta\omega_q{}^2}{\omega_q{}^2} = \frac{2W(q)}{N}\sum_{n=0}^{N} W(n)\cos^2\!\left(\frac{\pi qn}{N}\right)\frac{\delta\omega_{on}{}^2}{\omega_o{}^2}$$

For $\pi/2$ mode, this has the happy result that the cosine terms are all zero for odd n, so frequency errors in odd numbered (coupling) cells do not contribute to the frequency of the $\pi/2$ mode. The X^q are also perturbed by the frequency errors;

$$\delta X^q{}_n = \sum_{r\neq q}\frac{W(r)}{2}\left(\frac{\varepsilon_{q+r} + \varepsilon_{q-r}}{\dfrac{\omega_q{}^2}{\omega_{or}{}^2} - 1}\right)\cos\frac{\pi nr}{N}$$

where $$\varepsilon_r = \frac{2}{N}\sum_{p=0}^{N} W(p)\,\frac{\delta\omega_{op}{}^2}{\omega_o{}^2}\cos\frac{\pi pr}{N}$$

is the Fourier transform of the errors. In the case of $\pi/2$ mode, this becomes

---

[1] D. E. Nagle, et. al., RSI 38,1583-1587, November, 1967

$$\delta X^{\pi/2}{}_n = \frac{1}{k} \sum_{p=1}^{N} \varepsilon_p \frac{\sin\frac{\pi pn}{N} \sin\frac{\pi n}{2}}{\sin\frac{\pi p}{N}}$$

It is seen that since $\sin(n\pi/2) = 0$ for even n, frequency errors do not perturb the field pattern of the $\pi/2$ mode, at least to first order.

Application to DAW Structures

As contrasted with the SCS, which has distinct cavities, the DAW has two modes in the same physical cavities as, for example, has the post couplers in the Alvarez structure. Then how is one to define the "cells" to be used in perturbation theory? One approach, that taken here, is to consider how one might construct cavities which would support the two modes; TM_{010} (c) for the coupling mode (odd numbered cavities) and TM_{020} (a) for the accelerating (even numbered cavities) for the normal operation. Perusal of the field patterns for the two modes, shown in figure 1, shows that closing the cavity with radial metal screens at the washers will short out the TM_{020} but support the TM_{010}, while closing at the disc will short out the TM_{010} and support the TM_{020}. Indeed SAIC has supplied cavity endplates to do exactly this for investigation of the two modes. Then the equivalent cavity chain to represent the normal DAW is: 1/2a* (endplate to washer), c* (disk to disk), a (washer to washer), c (disk to disk), a, c,......c*, 1/2a*. in the interior of the cavity, it is seen (see figure 2) that every coupling cell is perturbed by 1/2 stem (frequency f_{cp}) while the end coupling cells (n=1 and n=N-1)are perturbed further by the endplate (f^*_{cp}). Accelerating cells in the interior are alternately unperturbed (f_{au} for n/2 even), and perturbed (f_{ap} for n/2 odd) by the stems. In addition, the end half (a) cells are perturbed by the hole in the endplate, although we will not consider this effect here.

Model Cavities

Three short cavity configurations were provided (see figure 3); (1_u) two coupling endplates are assembled as a cavity giving frequency f_{1c} ,(1_p) full stems are installed in this cavity giving frequency f_{1cpt}, (2) a "two cell" configuration employs (a) end plates and one pair of washers supported by stems giving frequency f_{2a}, (3) a "three cell" configuration employs coupling endplates on the same cylinder with two washers giving frequency f_{3c}, and (4) a "ten cell" configuration has five pairs of stem-supported washers with (a) endplates giving frequency f_{10}. The ten cell cavity could also be terminated with (c) endplates, but this has not been done. We can use linearized perturbation theory to interpret the results and implications.

(1_u) one cell, no stems

n	W(n)	f_n	$\cos^2$
0	1/2	f_{cu}	1
1	1	f_{au}	0
2	1/2	f_{cu}	1

$f_{1c} = 2f_{cu}/2 + 0 \cdot f_{au} = f_{cu}$. This is the unperturbed coupling mode frequency.

(1_p) one cell, full stems

n	W(n)	f_n	$\cos^2$
0	1/2	f_{cp2}	1
1	1	f_{au}	0
2	1/2	f_{cp2}	1

$f_{1cpt} = 2f_{cp2}/2 + 0 \cdot f_{au} = f_{cp2}$. This is the doubly perturbed (full stem) coupling mode frequency. The normal cell coupling mode frequency, f_{cp}, perturbed by half a stem, is given by the mean of the previous frequencies $\quad f_{cp} = (f_{1c} + f_{1cpt})/2$.

(2) two cell cavity

n	W(n)	f_n	$\cos^2$
0	1/2	f_{au}	1
1	1	f^*_{cp}	0
2	1	f_{ap}	1
3	1	f^*_{cp}	0
4	1/2	f_{au}	1

$f_{2a} = (2 \cdot f_{au}/2 + f_{ap} + 2 \cdot 0 \cdot f^*_{cp})/2 = (f_{au} + f_{ap})/2$

(3) three cell cavity

n	W(n)	f_n	$\cos^2$
0	1/2	f_{cu}	1
1	1	f_{au}	0
2	1	f_{cp}	1
3	1	f_{ap}	0
4	1	f_{cp}	1
5	1	f_{au}	0
6	1/2	f_{cu}	1

$f_{3c} = (f_{cu} + 2 \cdot f_{cp})/3$

(4) ten cell cavity, (a) termination

n	W(n)	f_n	$\cos^2$
0	1/2	f_{au}	1
1	1	f^*_{cp}	0
2	1	f_{ap}	1
3	1	f_{cp}	0
4	1	f_{au}	1
***			
18	1	f_{ap}	1
19	0	f^*_{cp}	0
20	1/2	f_{au}	1

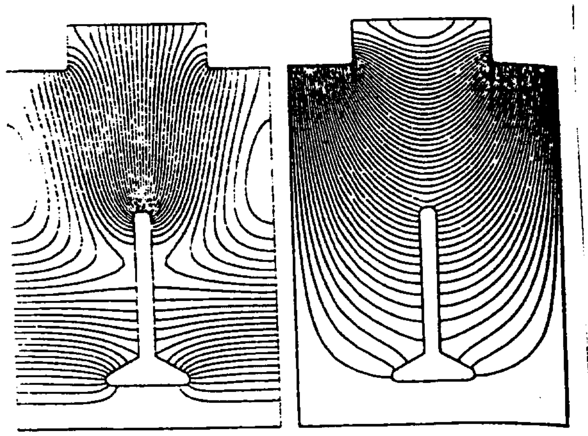
$f_{10} = (2 \cdot f_{au}/2 + 4 \cdot f_{au} + 5 \cdot f_{ap})/10 = (f_{ap} + f_{au})/2$

Measurement results

f_{1c}	782.9361
f_{1cpt}	826.7761
f_{2a}	796.8417
f_{3c}	798.0418

We can draw some conclusions from these results. First, the two cell cavity accurately predicts the ten cell frequency. This indeed turned out to be true, although the cavity dimensions were altered to get to 805 MHz. Second, the determination of the perturbed coupling mode was redundant, allowing a consistency check. Using f_{1c} and f_{1cpt} to estimate f_{cp} yields 804.8561 MHz, while using f_{1c} and f_{3c} yields 805.5947. This is probably satisfactory agreement.

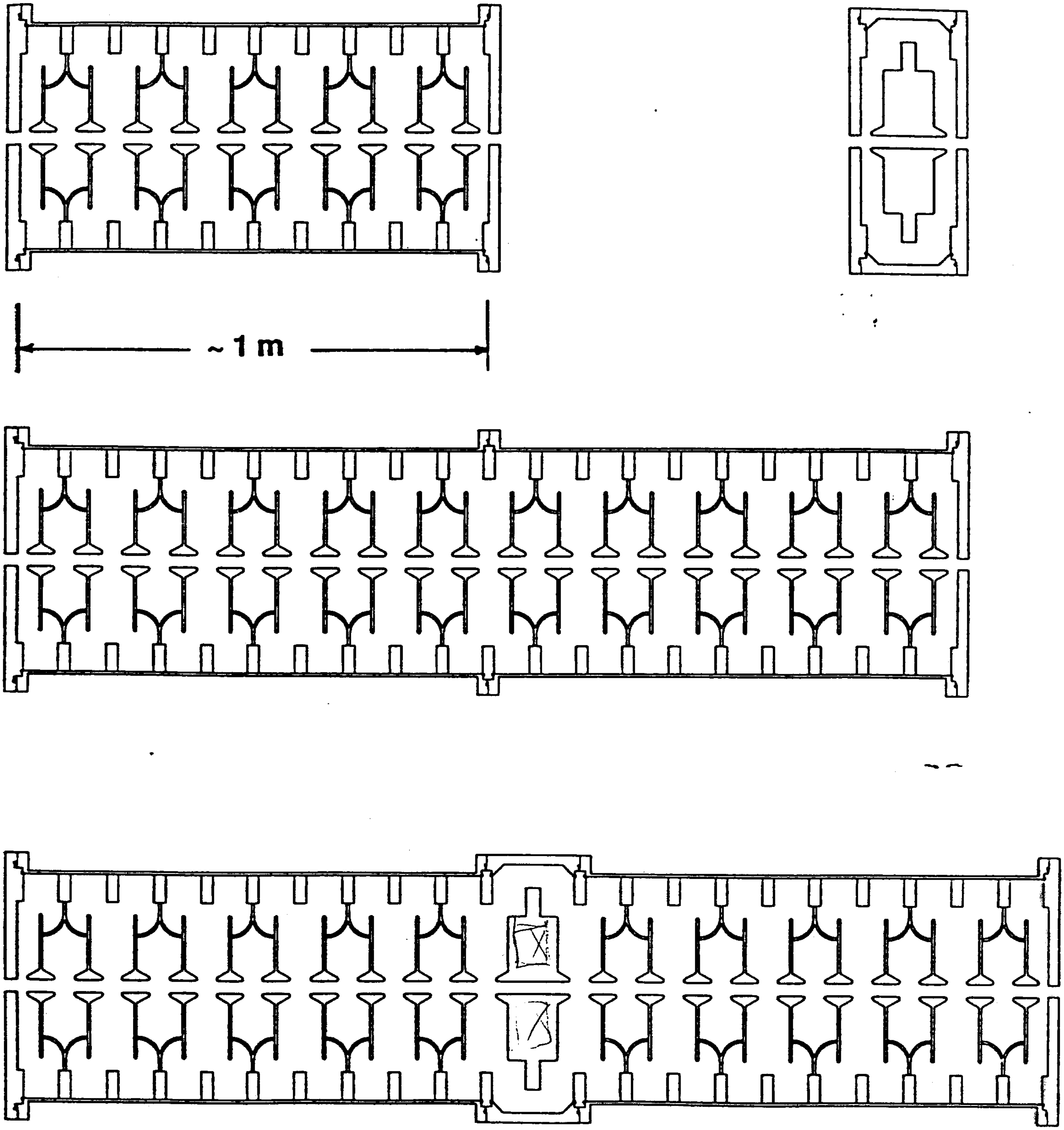
There are some omissions in the data. (A) It would be useful to know the two (a) frequencies separately, even though such knowledge is not necessary for the operating frequency of the final result. (B) Perhaps as measurements become more refined and the dimensional tolerances of the cavities becomes better understood, the two cell and ten cell could be used to measure the effect of the hole on the (a) mode at the endplates. (C) measuring the dispersion curve of the two cell and ten cell might tell something about the perturbation to the coupling mode in the end coupling cells (f^*_{cp}). It might also tell us the magnitude of the coupling coefficient. This presumably would allow us to estimate the magnitude of the excitation of the coupling mode in the ten cell, and what effect this has on the Q of the ten cell. (D) It is not completely clear to me that the left and right coupling coefficients, say as viewed from a coupling cell, are equal. Understanding this will require a new perturbation theory, although I believe Bo Si Wang has obtained such a result.



Electric field of the accelerating mode (left) and that of the coupling mode (right).

Figure 1

Disk and Washer (DAW) Linac Structure



Accelerating Mode Termination

Figure 2

ONE, TWO AND THREE CELL FREQUENCY TEST

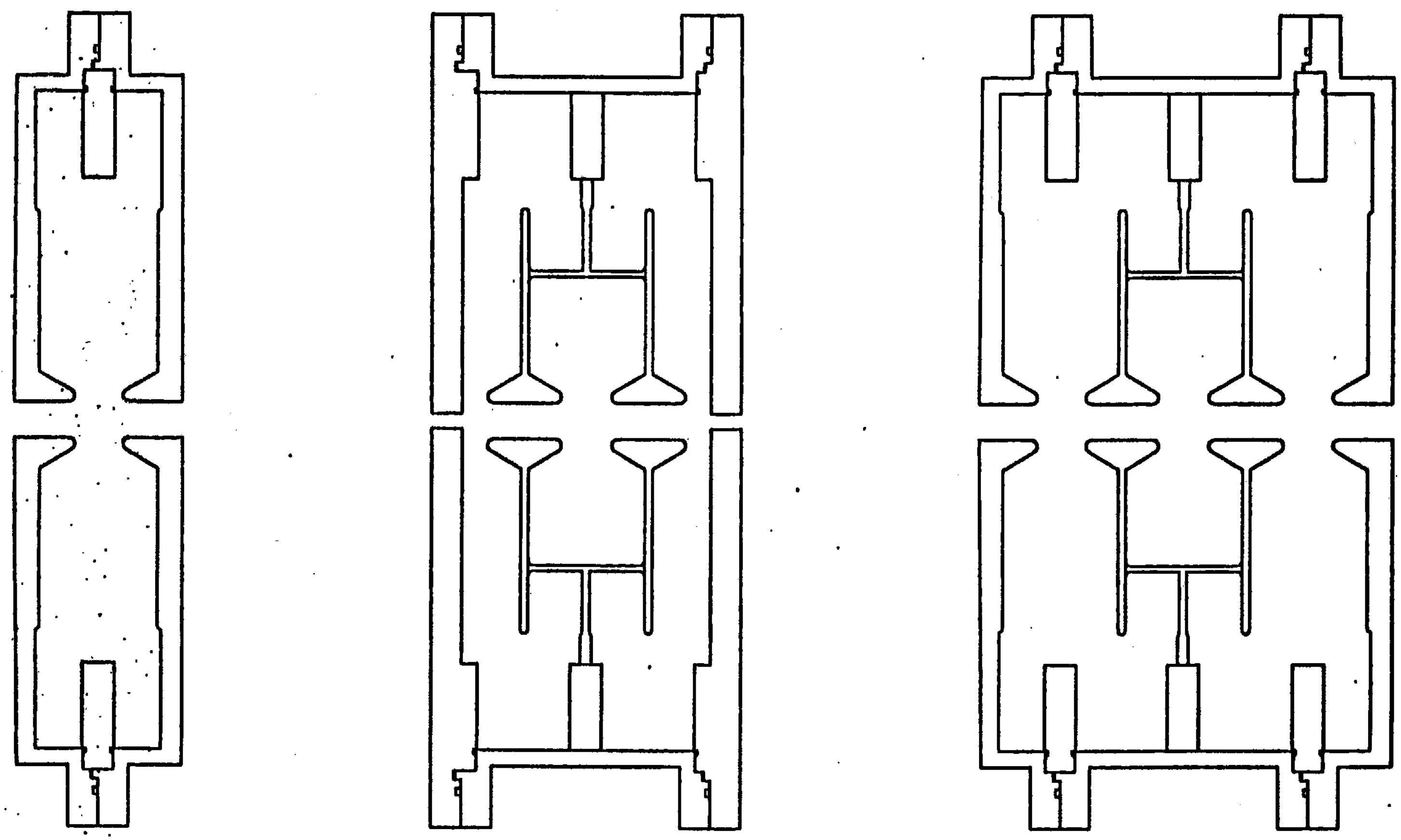


Figure 3